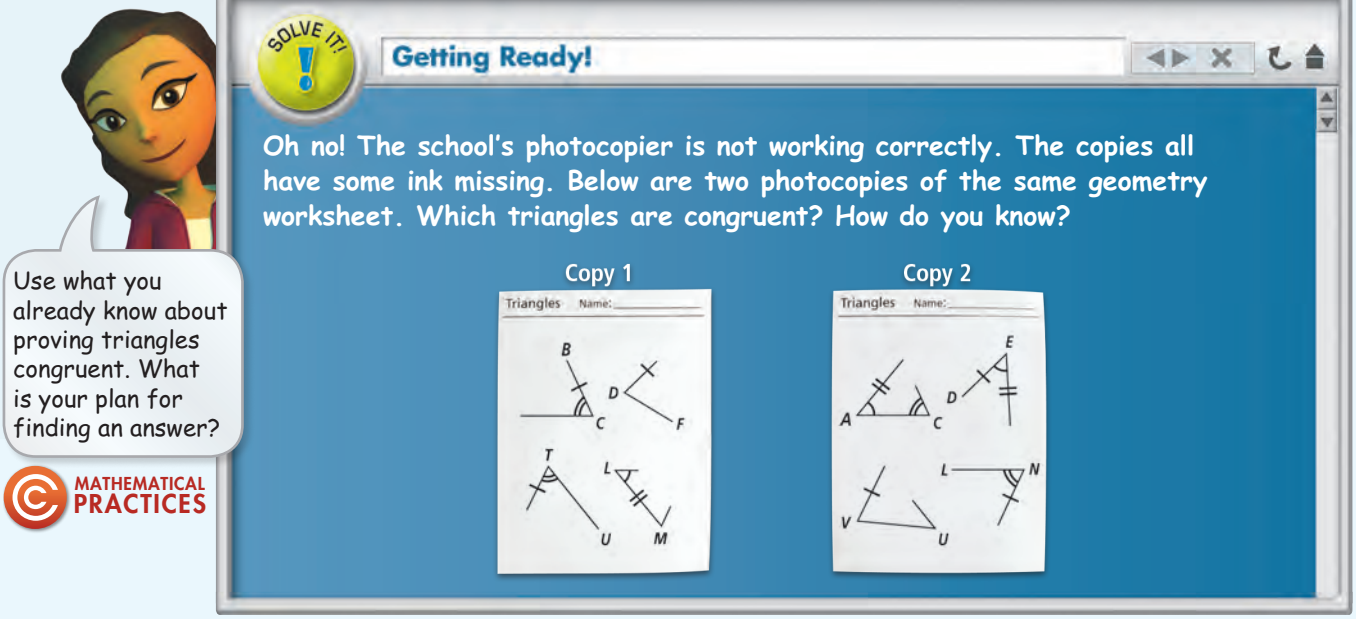
4-3 Triangle Congruence by ASA and AAS

Common Core State Standards

MATHEMATICAL PRACTICES MP 1, MP 3, MP 7

Objective To prove two triangles congruent using the ASA Postulate and the AAS Theorem



You already know that triangles are congruent if two pairs of sides and the included angles are congruent (SAS). You can also prove triangles congruent using other groupings of angles and sides.

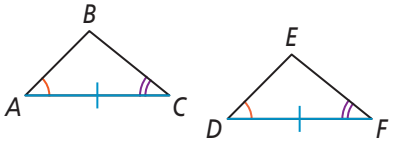
Essential Understanding You can prove that two triangles are congruent without having to show that *all* corresponding parts are congruent. In this lesson, you will prove triangles congruent by using one pair of corresponding sides and two pairs of corresponding angles.

take note

Postulate 4-3 Angle-Side-Angle (ASA) Postulate

Postulate
If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.

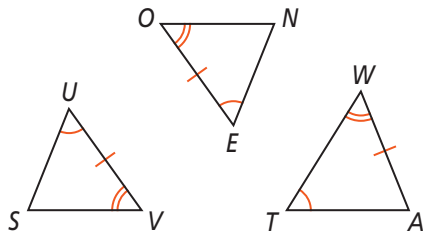
If . . .
$\angle A \cong \angle D$, $\overline{AC} \cong \overline{DF}$, $\angle C \cong \angle F$



Then . . .
$\triangle ABC \cong \triangle DEF$

Problem 1 Using ASA

Which two triangles are congruent by ASA? Explain.



Know

From the diagram you know
- $\angle U \cong \angle E \cong \angle T$
- $\angle V \cong \angle O \cong \angle W$
- $\overline{UV} \cong \overline{EO} \cong \overline{AW}$

Need

To use ASA, you need two pairs of congruent angles and a pair of included congruent sides.

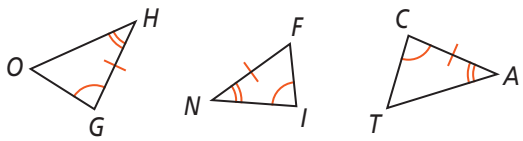
Plan

You already have pairs of congruent angles. So, identify the included side for each triangle and see whether it has a congruence marking.

In $\triangle SUV$, $\overline{UV}$ is included between $\angle U$ and $\angle V$ and has a congruence marking. In $\triangle NEO$, $\overline{EO}$ is included between $\angle E$ and $\angle O$ and has a congruence marking. In $\triangle ATW$, $\overline{TW}$ is included between $\angle T$ and $\angle W$ but does *not* have a congruence marking.

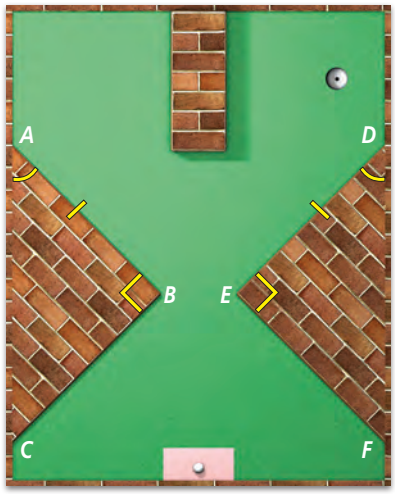
Since $\angle U \cong \angle E$, $\overline{UV} \cong \overline{EO}$, and $\angle V \cong \angle O$, $\triangle SUV \cong \triangle NEO$.

Got It? **1.** Which two triangles are congruent by ASA? Explain.



Proof Problem 2 Writing a Proof Using ASA

Recreation **Members of a teen organization are building a miniature golf course at your town's youth center. The design plan calls for the first hole to have two congruent triangular bumpers. Prove that the bumpers on the first hole, shown at the right, meet the conditions of the plan.**



Plan

Can you use a plan similar to the plan in Problem 1?
Yes. Use the diagram to identify the included side for the marked angles in each triangle.

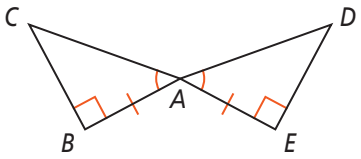
Given: $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, $\angle B$ and $\angle E$ are right angles

Prove: $\triangle ABC \cong \triangle DEF$

Proof: $\angle B \cong \angle E$ because all right angles are congruent, and you are given that $\angle A \cong \angle D$. $\overline{AB}$ and $\overline{DE}$ are included sides between the two pairs of congruent angles. You are given that $\overline{AB} \cong \overline{DE}$. Thus, $\triangle ABC \cong \triangle DEF$ by ASA.

Got It? **2. Given:** $\angle CAB \cong \angle DAE$, $\overline{BA} \cong \overline{EA}$, $\angle B$ and $\angle E$ are right angles

Prove: $\triangle ABC \cong \triangle AED$



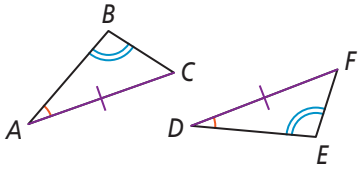
You can also prove triangles congruent by using two angles and a nonincluded side, as stated in the theorem below.

take note

Theorem 4-2 Angle-Angle-Side (AAS) Theorem

Theorem
If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of another triangle, then the triangles are congruent.

If . . .
$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\overline{AC} \cong \overline{DF}$

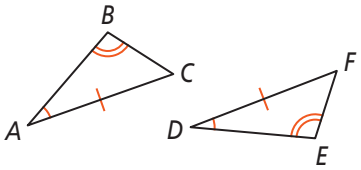


Then . . .
$\triangle ABC \cong \triangle DEF$

Proof **Proof of Theorem 4-2: Angle-Angle-Side Theorem**

Given: $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\overline{AC} \cong \overline{DF}$

Prove: $\triangle ABC \cong \triangle DEF$



$\angle A \cong \angle D$ — Given

$\angle B \cong \angle E$ — Given

$\overline{AC} \cong \overline{DF}$ — Given

$\angle C \cong \angle F$ — Third Angles Theorem

$\triangle ABC \cong \triangle DEF$ — ASA

You have seen and used three methods of proof in this book—two-column, paragraph, and flow proof. Each method is equally as valid as the others. Unless told otherwise, you can choose any of the three methods to write a proof. Just be sure your proof always presents logical reasoning with justification.

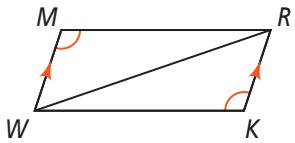
Problem 3 Writing a Proof Using AAS

Plan

How does information about parallel sides help?
You will need another pair of congruent angles to use AAS. Think back to what you learned in Chapter 3. $\overline{WR}$ is a transversal here.

Given: $\angle M \cong \angle K$, $\overline{WM} \parallel \overline{RK}$

Prove: $\triangle WMR \cong \triangle RKW$

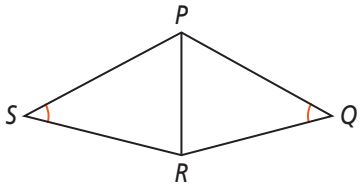


Statements	Reasons
1) $\angle M \cong \angle K$	1) Given
2) $\overline{WM} \parallel \overline{RK}$	2) Given
3) $\angle MWR \cong \angle KRW$	3) If lines are $\parallel$, then alternate interior $\measuredangle$ are $\cong$.
4) $\overline{WR} \cong \overline{WR}$	4) Reflexive Property of Congruence
5) $\triangle WMR \cong \triangle RKW$	5) AAS

Got It? **3. a. Given:** $\angle S \cong \angle Q$, $\overline{RP}$ bisects $\angle SRQ$

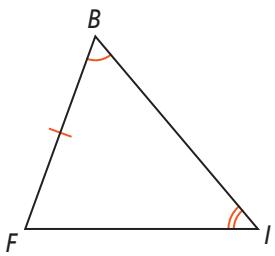
Prove: $\triangle SRP \cong \triangle QRP$

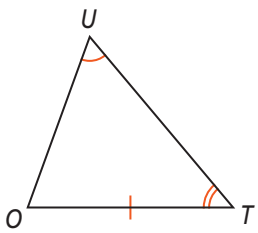
b. Reasoning In Problem 3, how could you prove that $\triangle WMR \cong \triangle RKW$ by ASA? Explain.



Problem 4 Determining Whether Triangles Are Congruent

Multiple Choice **Use the diagram at the right. Which of the following statements best represents the answer and justification to the question, "Is $\triangle BIF \cong \triangle UTO$?"**





(A) Yes, the triangles are congruent by ASA.

(B) No, $\overline{FB}$ and $\overline{OT}$ are not corresponding sides.

(C) Yes, the triangles are congruent by AAS.

(D) No, $\angle B$ and $\angle U$ are not corresponding angles.

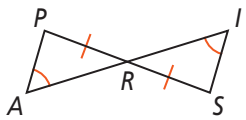
Think

Can you eliminate any of the choices?
Yes. If $\triangle BIF \cong \triangle UTO$ then $\angle B$ and $\angle U$ would be corresponding angles. You can eliminate choice D.

The diagram shows that two pairs of angles and one pair of sides are congruent. The third pair of angles is congruent by the Third Angles Theorem. To prove these triangles congruent, you need to satisfy ASA or AAS.

ASA and AAS both fail because $\overline{FB}$ and $\overline{TO}$ are not included between the same pair of congruent corresponding angles, so they are not corresponding sides. The triangles are not necessarily congruent. The correct answer is B.

Got It? **4.** Are $\triangle PAR$ and $\triangle SIR$ congruent? Explain.



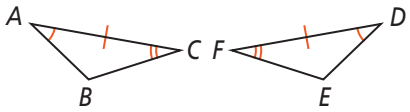
Lesson Check

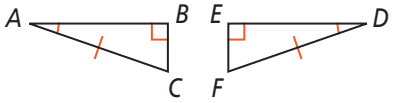
Do you know HOW?

1. In $\triangle RST$, which side is included between $\angle R$ and $\angle S$?

2. In $\triangle NOM$, $\overline{NO}$ is included between which angles?

Which postulate or theorem could you use to prove $\triangle ABC \cong \triangle DEF$?

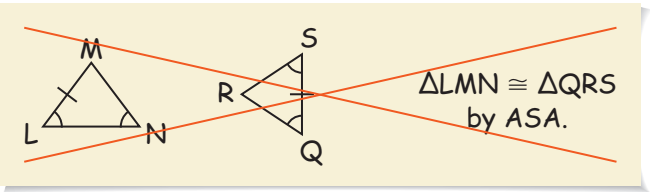
3. 

4. 

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. **Compare and Contrast** How are the ASA Postulate and the SAS Postulate alike? How are they different?

6. **Error Analysis** Your friend asks you for help on a geometry exercise. Below is your friend's paper. What error did your friend make? Explain.

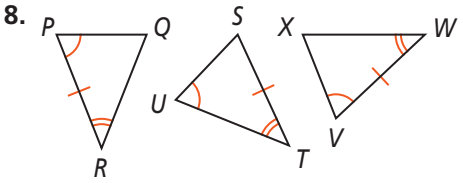


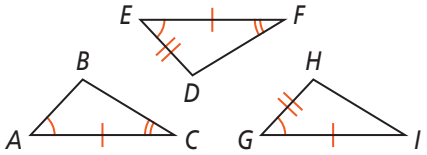
7. **Reasoning** Suppose $\angle E \cong \angle I$ and $\overline{FE} \cong \overline{GI}$. What else must you know in order to prove $\triangle FDE \cong \triangle GHI$ by ASA? By AAS?

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Name two triangles that are congruent by ASA. See Problem 1.

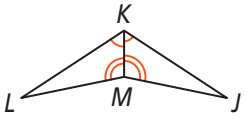
8. 

9. 

10. **Developing Proof** Complete the paragraph proof by filling in the blanks. See Problem 2.

Given: $\angle LKM \cong \angle JKM$, $\angle LMK \cong \angle JMK$

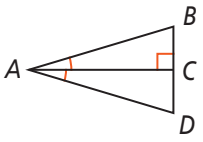
Prove: $\triangle LKM \cong \triangle JKM$



Proof: $\angle LKM \cong \angle JKM$ and $\angle LMK \cong \angle JMK$ are given. $\overline{KM} \cong \overline{KM}$ by the **a.** ? Property of Congruence. So, $\triangle LKM \cong \triangle JKM$ by **b.** ?.

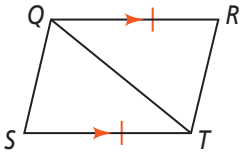
11. Proof **Given:** $\angle BAC \cong \angle DAC$, $\overline{AC} \perp \overline{BD}$

Prove: $\triangle ABC \cong \triangle ADC$



12. Proof **Given:** $\overline{QR} \cong \overline{TS}$, $\overline{QR} \parallel \overline{TS}$

Prove: $\triangle QRT \cong \triangle TSQ$

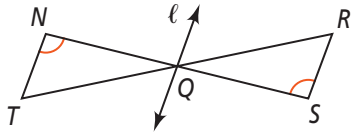


13. Developing Proof Complete the two-column proof by filling in the blanks.

See Problem 3.

Given: $\angle N \cong \angle S$, line ℓ bisects $\overline{TR}$ at Q

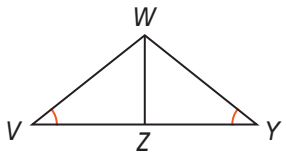
Prove: $\triangle NQT \cong \triangle SQR$



Statements	Reasons
1) $\angle N \cong \angle S$	1) Given
2) $\angle NQT \cong \angle SQR$	2) a. ?
3) Line ℓ bisects $\overline{TR}$ at Q.	3) b. ?
4) c. ?	4) Definition of bisect
5) $\triangle NQT \cong \triangle SQR$	5) d. ?

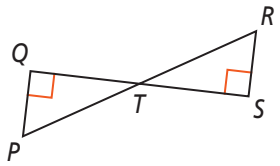
14. Proof **Given:** $\angle V \cong \angle Y$, $\overline{WZ}$ bisects $\angle VWY$

Prove: $\triangle VWZ \cong \triangle YWZ$



15. Proof **Given:** $\overline{PQ} \perp \overline{QS}$, $\overline{RS} \perp \overline{SQ}$, T is the midpoint of $\overline{PR}$

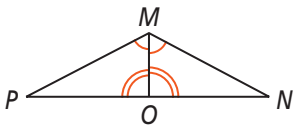
Prove: $\triangle PQT \cong \triangle RST$



Determine whether the triangles must be congruent. If so, name the postulate or theorem that justifies your answer. If not, explain.

See Problem 4.

16.



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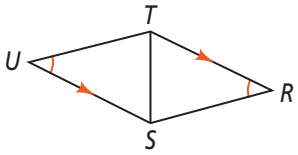

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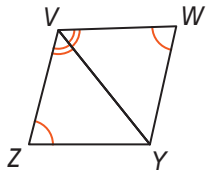
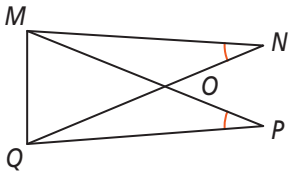


B Apply

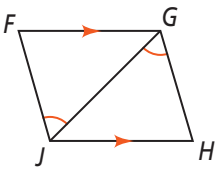
19. Proof **Given:** $\angle N \cong \angle P$, $\overline{MO} \cong \overline{QO}$

Prove: $\triangle MON \cong \triangle QOP$



20. Proof **Given:** $\angle FJG \cong \angle HGJ$, $\overline{FG} \parallel \overline{JH}$

Prove: $\triangle FGJ \cong \triangle HJG$



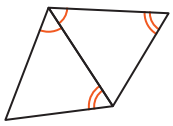
21. Think About a Plan While helping your family clean out the attic, you find the piece of paper shown at the right. The paper contains clues to locate a time capsule buried in your backyard. The maple tree is due east of the oak tree in your backyard. Will the clues always lead you to the correct spot? Explain.

- How can you use a diagram to help you?
- What type of geometric figure do the paths and the marked line form?
- How does the position of the marked line relate to the positions of the angles?

Mark a line on the ground from the oak tree to the maple tree. From the oak tree, walk along a path that forms a 70° angle with the marked line, keeping the maple tree to your right. From the maple tree, walk along a path that forms a 40° angle with the marked line. The time capsule is buried where the paths meet.

22. Constructions Use a straightedge to draw a triangle. Label it $\triangle JKL$. Construct $\triangle MNP \cong \triangle JKL$ so that the triangles are congruent by ASA.

23. Reasoning Can you prove that the triangles at the right are congruent? Justify your answer.

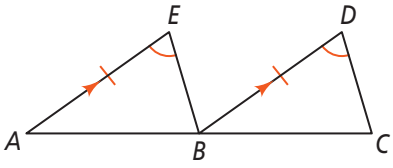


24. Writing Anita says that you can rewrite any proof that uses the AAS Theorem as a proof that uses the ASA Postulate. Do you agree with Anita? Explain.

25. Proof

Given: $\overline{AE} \parallel \overline{BD}$, $\overline{AE} \cong \overline{BD}$, $\angle E \cong \angle D$

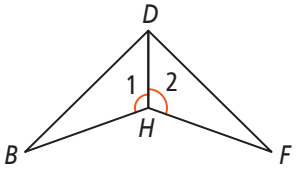
Prove: $\triangle AEB \cong \triangle BDC$



26. Proof

Given: $\angle 1 \cong \angle 2$, and $\overline{DH}$ bisects $\angle BDF$.

Prove: $\triangle BDH \cong \triangle FDH$

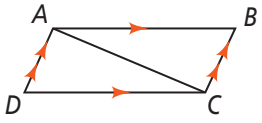


27. Draw a Diagram Draw two noncongruent triangles that have two pairs of congruent angles and one pair of congruent sides.

28. Proof

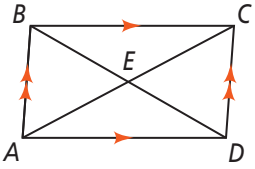
Given: $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$

Prove: $\triangle ABC \cong \triangle CDA$

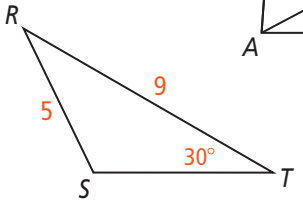


Challenge

29. Given $\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{DC}$, name as many pairs of congruent triangles as you can.



30. Constructions In $\triangle RST$ at the right, $RS = 5$, $RT = 9$, and $m\angle T = 30$. Show that there is no SSA congruence rule by constructing $\triangle UVW$ with $UV = RS$, $UW = RT$, and $m\angle W = m\angle T$, but with $\triangle UVW \ncong \triangle RST$.

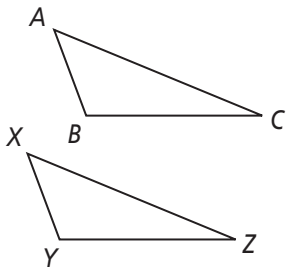


31. Probability Below are six statements about the triangles at the right.

$\angle A \cong \angle X$ $\quad \angle B \cong \angle Y$ $\quad \angle C \cong \angle Z$

$\overline{AB} \cong \overline{XY}$ $\quad \overline{AC} \cong \overline{XZ}$ $\quad \overline{BC} \cong \overline{YZ}$

There are 20 ways to choose a group of three statements from these six. What is the probability that three statements chosen at random from the six will guarantee that the triangles are congruent?



Standardized Test Prep

SAT/ACT

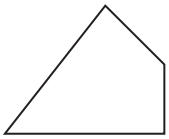
32. Suppose $\overline{RT} \cong \overline{ND}$ and $\angle R \cong \angle N$. What additional information do you need to prove that $\triangle RTJ \cong \triangle NDF$ by ASA?

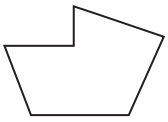
(A) $\angle T \cong \angle D$ (B) $\angle J \cong \angle F$ (C) $\angle J \cong \angle D$ (D) $\angle T \cong \angle F$

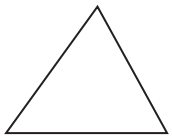
33. You plan to make a 2 ft-by-3 ft rectangular poster of class trip photos. Each photo is a 4 in.-by-6 in. rectangle. If the photos do not overlap, what is the greatest number of photos you can fit on your poster?

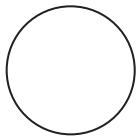
(F) 4 (G) 24 (H) 32 (I) 36

34. Which of the following figures is a concave polygon?

(A) 

(B) 

(C) 

(D) 

Short Response

35. Write the converse of the true conditional statement below. Then determine whether the converse is true or false.

If you are less than 18 years old, then you are too young to vote in the United States.

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 3

Look back at the information given on page 217 about how Jamal located the points in the diagram. In the Apply What You've Learned in Lesson 4-1, you copied the diagram, labeled it with the given information, and identified congruent sides and angles.

a. Look at the diagram you labeled. Which congruence postulate or theorem can you use to prove the two triangles are congruent?

b. Write a proof that the two triangles are congruent using only the information that you already have.